Exercício 10. Calcule $2^{\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \log_5 6}$

$\log_2 3 = \dfrac{\log 3}{\log 2}$ $\log_4 5 = \dfrac{\log 5}{\log 4}$

$\log_3 4 = \dfrac{\log 4}{\log 3}$ $\log_5 6 = \dfrac{\log 6}{\log 5}$

$\therefore \log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \log_5 6$

$= \dfrac{\cancel{\log 3}}{\log 2} \cdot \dfrac{\cancel{\log 4}}{\cancel{\log 3}} \cdot \dfrac{\cancel{\log 5}}{\cancel{\log 4}} \cdot \dfrac{\log 6}{\cancel{\log 5}} = \dfrac{\log 6}{\log 2} = \log_2 6$

Portanto,

$2^{\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \log_5 6} = 2^{\log_2 6} = 6.$

Exercício 5. Calcule o valor das expressões:

a) $\log 10 + \log_3 3^2$.

$\log 6 = \log_{10} 6$

b) $\log 6 \cdot \log_6 12 \cdot \log_{12} 10$.

$= \cancel{\log 6} \cdot \dfrac{\cancel{\log 12}}{\cancel{\log 6}} \cdot \dfrac{\log 10}{\cancel{\log 12}} = \log 10 = 1$

Exercício 12. Se o crescimento de uma população é de 20% ao ano, determine em quanto tempo essa população dobrará de tamanho. (Utilize $\log 2 = 0,3$ e $\log 3 = 0,48$)

t	0	1	2	3
$P(t)$	P_0	$P_0 + \frac{20}{100} \cdot P_0 = P_0 + 0,2P_0 = P_0(1+0,2) = 1,2^1 P_0$	$1,2P_0 + \frac{20}{100} \cdot 1,2P_0 = 1,2P_0(1+0,2) = 1,2P_0 \cdot 1,2 = (1,2)^2 P_0$	$(1,2)^2 P_0 + \frac{20}{100} \cdot (1,2)^2 P_0 = (1,2)^2 P_0 (1+0,2) = (1,2)^3 P_0$

$$\therefore P(t) = (1,2)^t P_0$$

Queremos t tal que:

$$P(t) = 2P_0 \Rightarrow (1,2)^t \cancel{P_0} = 2\cancel{P_0} \Rightarrow (1,2)^t = 2$$

$$\Rightarrow \left(\frac{12}{10}\right)^t = 2 \Rightarrow t = \log_{\frac{12}{10}} 2 = \frac{\log 2}{\log\left(\frac{12}{10}\right)}$$

$$\Rightarrow t = \frac{\log 2}{\log 12 - \cancel{\log 10}^{1}} = \frac{\log 2}{\log(4 \cdot 3) - 1} = \frac{\log 2}{\log 4 + \log 3 - 1}$$

$$\Rightarrow t = \frac{\log 2}{\log(2^2) + \log 3 - 1} = \frac{\log 2}{2\log 2 + \log 3 - 1} = \frac{0,3}{2 \cdot 0,3 + 0,48 - 1} = \frac{0,3}{0,08}$$

$$= 3,75 = 3 \text{ anos e } 9 \text{ meses}.$$

$$13\% = \frac{13}{100} = 0{,}13$$

t	0	1	2
d	100	$100 + 0{,}13 \cdot 100$ $\parallel$ $100(1+0{,}13)^{1}$ $\parallel$ $1{,}13 \cdot 100$	$\underbrace{100(1+0{,}13)} + 0{,}13\,\underbrace{(1+0{,}13)\cdot 100}$ $\parallel$ $100(1+0{,}13)\cdot(1+0{,}13)$ $\parallel$ $100(1+0{,}13)^{2}$

$$\therefore\ d(t) = \underbrace{100}\,(1+\underbrace{0{,}13})^{t}$$

$$\boxed{d(t) = d_0\,(1+j)^{t}}$$